Ex 7

Let R_1 and R_2 be the "congruent modulo 3" and the "congruent modulo 4" relations, respectively, on the set of integers. That is, $R_1 = \{(a, b) \mid a \equiv b \pmod{3}\}$ and $R_2 = \{(a, b) \mid a \equiv b \pmod{4}\}$. Find

a) $R_1 \cup R_2$.

b) $R_1 \cap R_2$.

c) $R_1 - R_2$.

d) $R_2 - R_1$.

e) $R_1 \oplus R_2$.

SOLUTION

$$R_1 = \{(a,b)|a \equiv b \pmod 3\}$$

$$R_2 = \{(a,b)|a \equiv b \pmod 4\}$$

(a) The union of two relations contains all ordered pairs that are in either relation.

$$\begin{aligned} R_1 \cup R_2 &= \{(a,b)|a \equiv b \pmod 3 \text{ or } a \equiv b \pmod 4\} \\ &= \{(a,b)|a-b \equiv 0 \pmod 3 \text{ or } a-b \equiv 0 \pmod 4\} \\ &= \{(a,b)|[a-b \equiv 0,3,6 \text{ or } 9 \pmod{12}] \text{ or } [a-b \equiv 0,4,8 \pmod{12}]\} \\ &= \{(a,b)|[a-b \equiv 0,3,4,6,8 \text{ or } 9 \pmod{12}]\} \end{aligned}$$

Note: a is a multiple of b if and only if b divides a.

(b) The intersection of two relations contains all ordered pairs that are in both relations.

$$\begin{aligned} R_1 \cap R_2 &= \{(a,b) | a \equiv b \pmod 3 \text{ and } a \equiv b \pmod 4\} \\ &= \{(a,b) | a \equiv b \pmod{12}\} \end{aligned}$$

(c) $R_1 - R_2$ contains all ordered pairs that are in the relation R_1 that do not occur in the relation R_2.

$$\begin{aligned} R_1 - R_2 &= \{(a,b) | a \equiv b \pmod 3 \text{ and } \neg[a \equiv b \pmod 4]\} \\ &= \{(a,b) | a - b \equiv 0 \pmod 3 \text{ and } \neg[a - b \equiv 0 \pmod 4]\} \\ &= \{(a,b) | [a - b \equiv 0, 3, 6 \text{ or } 9 \pmod{12}] \text{ and } \neg[a - b \equiv 0, 4, 8 \pmod{12}]\} \\ &= \{(a,b) | a - b \equiv 3, 6 \text{ or } 9 \pmod{12}\} \end{aligned}$$

(d) $R_2 - R_1$ contains all ordered pairs that are in the relation R_2 that do not occur in the relation R_1.

$$\begin{aligned} R_2 - R_1 &= \{(a,b) | a \equiv b \pmod 4 \text{ and } \neg[a \equiv b \pmod 3]\} \\ &= \{(a,b) | a - b \equiv 0 \pmod 4 \text{ and } \neg[a - b \equiv 0 \pmod 3]\} \\ &= \{(a,b) | [a - b \equiv 0,4,8 \pmod{12}] \text{ and } \neg[a - b \equiv 0,3,6 \text{ or } 9 \pmod{12}]\} \\ &= \{(a,b) | a - b \equiv 4 \text{ or } 8 \pmod{12}\} \end{aligned}$$

(e) $R_1 \oplus R_2$ contains all ordered pairs that are in the relation R_1 or R_2, but not in both.

$$\begin{aligned} R_1 \oplus R_2 &= \{(a,b) | [a \equiv b \pmod 3 \text{ or } a \equiv b \pmod 4] \text{ and } \neg[a \equiv b \pmod{12}]\} \\ &= \{(a,b) | [a - b \equiv 0 \pmod 3 \text{ or } a - b \equiv 0 \pmod 4] \text{ and } \neg[a - b \equiv 0 \pmod{12}]\} \\ &= \{(a,b) | ([a - b \equiv 0,3,6 \text{ or } 9 \pmod{12}] \text{ or } [a - b \equiv 0,4,8 \pmod{12}]) \text{ and } \neg[a - b \equiv 0 \pmod{12}]\} \\ &= \{(a,b) | [a - b \equiv 3,4,6,8 \text{ or } 9 \pmod{12}]\} \end{aligned}$$

Ex 8

Let R be a relation that is reflexive and transitive. Prove that $R^n = R$ for all positive integers n.

Let R be the relation on the set $\{1, 2, 3, 4, 5\}$ containing the ordered pairs $(1, 1)$, $(1, 2)$, $(1, 3)$, $(2, 3)$, $(2, 4)$, $(3, 1)$, $(3, 4)$, $(3, 5)$, $(4, 2)$, $(4, 5)$, $(5, 1)$, $(5, 2)$, and $(5, 4)$. Find

a) R^2. **b)** R^3. **c)** R^4. **d)** R^5.

DEFINITIONS

A relation R on a set A is **reflexive** if $(a, a) \in R$ for every element $a \in A$.
A relation R on a set A is **transitive** if $(a, b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$

The **composite** $S \circ R$ consists of all ordered pairs (a, c) for which there exists an element b such that $(a, b) \in R$ and $(b, c) \in S$

SOLUTION

Given: Relation R on a set A is reflexive and transitive

To proof: $R^n = R$ for all positive integers n.

PROOF BY INDUCTION

Let $P(n)$ by the statement "$R^n = R$".

Basis case $n = 1$

The result is trivial, because $R = R$ is always true and $R^1 = R$.

Inductive case Let us assume that $P(n)$ be true.

$$R^n = R$$

We need to prove that $P(n+1)$ is also true.

$$\begin{aligned} R^{n+1} &= R^n \circ R \\ &= R \circ R \end{aligned}$$

Let $(a, b) \in R$.
Since R is reflexive: $(b, b) \in R$.
By the definition of composite: $(a, b) \in R \circ R = R^{n+1}$

$$R \subseteq R^{n+1}$$

Let $(a,b) \in R^{n+1} = R \circ R$.
By the definition of composite: $\exists c \in R : (a,c) \in R$ and $(c,b) \in R$
By the definition of transitive: $(a,b) \in R$

$$R^{n+1} \subseteq R$$

Since $R^{n+1} \subseteq R$ and $R \subseteq R^{n+1}$, we have then shown:

$$R^{n+1} = R$$

Thus $P(n+1)$ is true.

Conclusion By the principle of mathematical induction, $P(n)$ is true for all positive integers n.
□

$R^2 = (1,2)(1,3)(1,4)(1,1)(1,5)(2,1)(2,4)(2,5)(2,2)(3,2)(3,3)(3,4)(3,5)(3,1)(4,3)(4,4)(4,2)(4,1)(5,1)(5,2)(5,3)(5,4)(5,5)$

$R^3 = (1,2)(1,3)(1,4)(1,1)(1,5)(2,2)(2,3)(2,4)(2,5)(2,1)(3,2)(3,3)(3,4)(3,1)(3,5)(4,1)(4,4)(4,5)(4,2)(4,3)(5,2)(5,3)(5,4)(5,1)(5,5)$

$R^4 = (1,1)(1,2)(1,3)(1,4)(1,5)(2,1)(2,2)(2,3)(2,4)(2,5)(3,2)(3,3)(3,4)(3,5)(3,1)(4,3)(4,4)(4,2)(4,1)(4,5)(5,1)(5,2)(5,3)(5,4)(5,5)$

$R^5 = (1,2)(1,1)(1,3)(1,5)(1,4)(2,1)(2,2)(2,3)(2,4)(2,5)(3,2)(3,3)(3,4)(3,5)(3,1)(4,3)(4,4)(4,2)(4,1)(4,5)(5,1)(5,2)(5,3)(5,4)(5,5)$

Ex 9

Let R be the relation represented by the matrix

$$\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

Find the matrix representing

a) R^{-1}. **b)** $\overline{R}$. **c)** R^2.

DEFINITIONS

The **inverse relation** R^{-1} is the set $\{(b,a)|(a,b) \in R\}$
The **complementary relation** $\overline{R}$ is the set $\{(a,b)|(a,b) \notin R\}$
The **composite** $S \circ R$ consists of all ordered pairs (a,c) for which there exists an element b such that $(a,b) \in R$ and $(b,c) \in S$

A relation R can be represented by the matrix $\mathbf{M}_R = [m_{ij}]$

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R \end{cases}$$

SOLUTION

$$\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(a) The matrix corresponding to the inverse relation R^{-1} is the **transposed** of the matrix representing R:

$$\begin{aligned} \mathbf{M}_{R^{-1}} &= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}^T \\ &= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \\ &= \mathbf{M}_R \end{aligned}$$

(b) The matrix corresponding to the complementary relation $\overline{R}$ **changes every zero to a 1 and changes every one to a 0** in the matrix representing R:

$$\mathbf{M}_{\overline{R}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

(c) The matrix corresponding to the composite of two matrices is the Boolean product of two matrices.

$$\begin{aligned}
\mathbf{M}_{R^2} &= \mathbf{M}_R \odot \mathbf{M}_R \\
&= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} (0 \wedge 0) \vee (1 \wedge 1) \vee (1 \wedge 1) & (0 \wedge 1) \vee (1 \wedge 1) \vee (1 \wedge 0) & (0 \wedge 1) \vee (1 \wedge 0) \vee (1 \wedge 1) \\ (1 \wedge 0) \vee (1 \wedge 1) \vee (0 \wedge 1) & (1 \wedge 1) \vee (1 \wedge 1) \vee (0 \wedge 0) & (1 \wedge 1) \vee (1 \wedge 0) \vee (0 \wedge 1) \\ (1 \wedge 0) \vee (0 \wedge 1) \vee (1 \wedge 1) & (1 \wedge 1) \vee (0 \wedge 1) \vee (1 \wedge 0) & (1 \wedge 1) \vee (0 \wedge 0) \vee (1 \wedge 1) \end{bmatrix} \\
&= \begin{bmatrix} 0 \vee 1 \vee 1 & 0 \vee 1 \vee 0 & 0 \vee 0 \vee 1 \\ 0 \vee 1 \vee 0 & 1 \vee 1 \vee 0 & 1 \vee 0 \vee 0 \\ 0 \vee 0 \vee 1 & 1 \vee 0 \vee 0 & 1 \vee 0 \vee 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}
\end{aligned}$$

Ex 10

Let R_1 and R_2 be relations on a set A represented by the matrices

$$\mathbf{M}_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Find the matrices that represent

a) $R_1 \cup R_2$. **b)** $R_1 \cap R_2$. **c)** $R_2 \circ R_1$.
d) $R_1 \circ R_1$. **e)** $R_1 \oplus R_2$.

Ex 11

Let R be the relation represented by the matrix

$$\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

Find the matrices that represent

a) R^2. **b)** R^3. **c)** R^4.

SOLUTION

$$\mathbf{M}_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{M}_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(a) The matrix corresponding to the union of two relations is the **join** of the matrices representing each of the relations:

$$\begin{aligned} \mathbf{M}_{R_1 \cup R_2} &= \mathbf{M}_{R_1} \vee \mathbf{M}_{R_2} \\ &= \begin{bmatrix} 0 \vee 0 & 1 \vee 1 & 0 \vee 0 \\ 1 \vee 0 & 1 \vee 1 & 1 \vee 1 \\ 1 \vee 1 & 0 \vee 1 & 0 \vee 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{aligned}$$

(b) The matrix corresponding to the intersection of two relations is the **meet** of the matrices representing each of the relations:

$$\begin{aligned} \mathbf{M}_{R_1 \cap R_2} &= \mathbf{M}_{R_1} \wedge \mathbf{M}_{R_2} \\ &= \begin{bmatrix} 0 \wedge 0 & 1 \wedge 1 & 0 \wedge 0 \\ 1 \wedge 0 & 1 \wedge 1 & 1 \wedge 1 \\ 1 \wedge 1 & 0 \wedge 1 & 0 \wedge 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{aligned}$$

(c) The matrix corresponding to the composite of two matrices is the Boolean product of two matrices.

$$
\begin{aligned}
\mathbf{M}_{R_2 \circ R_1} &= \mathbf{M}_{R_1} \odot \mathbf{M}_{R_2} \\
&= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \\
&= \begin{bmatrix} (0 \wedge 0) \vee (1 \wedge 0) \vee (0 \wedge 1) & (0 \wedge 1) \vee (1 \wedge 1) \vee (0 \wedge 1) & (0 \wedge 0) \vee (1 \wedge 1) \vee (0 \wedge 1) \\ (1 \wedge 0) \vee (1 \wedge 0) \vee (1 \wedge 1) & (1 \wedge 1) \vee (1 \wedge 1) \vee (1 \wedge 1) & (1 \wedge 0) \vee (1 \wedge 1) \vee (1 \wedge 1) \\ (1 \wedge 0) \vee (0 \wedge 0) \vee (0 \wedge 1) & (1 \wedge 1) \vee (0 \wedge 1) \vee (0 \wedge 1) & (1 \wedge 0) \vee (0 \wedge 1) \vee (0 \wedge 1) \end{bmatrix} \\
&= \begin{bmatrix} 0 \vee 0 \vee 0 & 0 \vee 1 \vee 0 & 0 \vee 1 \vee 0 \\ 0 \vee 0 \vee 1 & 1 \vee 1 \vee 1 & 0 \vee 1 \vee 1 \\ 0 \vee 0 \vee 0 & 1 \vee 0 \vee 0 & 0 \vee 0 \vee 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}
\end{aligned}
$$

(d) The matrix corresponding to the composite of two matrices is the Boolean product of two matrices.

$$
\begin{aligned}
\mathbf{M}_{R_1 \circ R_1} &= \mathbf{M}_{R_1} \odot \mathbf{M}_{R_1} \\
&= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} (0 \wedge 0) \vee (1 \wedge 1) \vee (0 \wedge 1) & (0 \wedge 1) \vee (1 \wedge 1) \vee (0 \wedge 0) & (0 \wedge 0) \vee (1 \wedge 1) \vee (0 \wedge 0) \\ (1 \wedge 0) \vee (1 \wedge 1) \vee (1 \wedge 1) & (1 \wedge 1) \vee (1 \wedge 1) \vee (1 \wedge 0) & (1 \wedge 0) \vee (1 \wedge 1) \vee (1 \wedge 0) \\ (1 \wedge 0) \vee (0 \wedge 1) \vee (0 \wedge 1) & (1 \wedge 1) \vee (0 \wedge 1) \vee (0 \wedge 0) & (1 \wedge 0) \vee (0 \wedge 1) \vee (0 \wedge 0) \end{bmatrix} \\
&= \begin{bmatrix} 0 \vee 1 \vee 0 & 0 \vee 1 \vee 0 & 0 \vee 1 \vee 0 \\ 0 \vee 1 \vee 1 & 1 \vee 1 \vee 0 & 0 \vee 1 \vee 0 \\ 0 \vee 0 \vee 0 & 1 \vee 0 \vee 0 & 0 \vee 0 \vee 0 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}
\end{aligned}
$$

(e) The matrix corresponding to the union of two relations is the **join** of the matrices representing each of the relations:

$$
\begin{aligned}
\mathbf{M}_{R_1 \oplus R_2} &= \mathbf{M}_{R_1} \oplus \mathbf{M}_{R_2} \\
&= \begin{bmatrix} 0 \oplus 0 & 1 \oplus 1 & 0 \oplus 0 \\ 1 \oplus 0 & 1 \oplus 1 & 1 \oplus 1 \\ 1 \oplus 1 & 0 \oplus 1 & 0 \oplus 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}
\end{aligned}
$$

SOLUTION

$$\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(a) The matrix corresponding to the composite of two relations is the Boolean product of two corresponding matrices.

$$\begin{aligned}
\mathbf{M}_{R^2} &= \mathbf{M}_R \odot \mathbf{M}_R \\
&= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \\
&= \begin{bmatrix} (0\wedge 0)\vee(1\wedge 0)\vee(0\wedge 1) & (0\wedge 1)\vee(1\wedge 0)\vee(0\wedge 1) & (0\wedge 0)\vee(1\wedge 1)\vee(0\wedge 0) \\ (0\wedge 0)\vee(0\wedge 0)\vee(1\wedge 1) & (0\wedge 1)\vee(0\wedge 0)\vee(1\wedge 1) & (0\wedge 0)\vee(0\wedge 1)\vee(1\wedge 0) \\ (1\wedge 0)\vee(1\wedge 0)\vee(0\wedge 1) & (1\wedge 1)\vee(1\wedge 0)\vee(0\wedge 1) & (1\wedge 0)\vee(1\wedge 1)\vee(0\wedge 0) \end{bmatrix} \\
&= \begin{bmatrix} 0\vee 0\vee 0 & 0\vee 0\vee 0 & 0\vee 1\vee 0 \\ 0\vee 0\vee 1 & 0\vee 0\vee 1 & 0\vee 0\vee 0 \\ 0\vee 0\vee 0 & 1\vee 0\vee 0 & 0\vee 1\vee 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}
\end{aligned}$$

(b) The matrix corresponding to the composite of two relations is the Boolean product of two corresponding matrices.

$$\begin{aligned}
\mathbf{M}_{R^3} &= \mathbf{M}_{R^2} \odot \mathbf{M}_R \\
&= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \\
&= \begin{bmatrix} (0\wedge 0)\vee(0\wedge 0)\vee(1\wedge 1) & (0\wedge 1)\vee(0\wedge 0)\vee(1\wedge 1) & (0\wedge 0)\vee(0\wedge 1)\vee(1\wedge 0) \\ (1\wedge 0)\vee(1\wedge 0)\vee(0\wedge 1) & (1\wedge 1)\vee(1\wedge 0)\vee(0\wedge 1) & (1\wedge 0)\vee(1\wedge 1)\vee(0\wedge 0) \\ (0\wedge 0)\vee(1\wedge 0)\vee(1\wedge 1) & (0\wedge 1)\vee(1\wedge 0)\vee(1\wedge 1) & (0\wedge 0)\vee(1\wedge 1)\vee(1\wedge 0) \end{bmatrix} \\
&= \begin{bmatrix} 0\vee 0\vee 1 & 0\vee 0\vee 1 & 0\vee 0\vee 0 \\ 0\vee 0\vee 0 & 1\vee 0\vee 0 & 0\vee 1\vee 0 \\ 0\vee 0\vee 1 & 0\vee 0\vee 1 & 0\vee 1\vee 0 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}
\end{aligned}$$

(c) The matrix corresponding to the composite of two relations is the Boolean product of two corresponding matrices.

$$\mathbf{M}_{R^3} = \mathbf{M}_{R^2} \odot \mathbf{M}_R$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \wedge 0) \vee (1 \wedge 0) \vee (0 \wedge 1) & (1 \wedge 1) \vee (1 \wedge 0) \vee (0 \wedge 1) & (1 \wedge 0) \vee (1 \wedge 1) \vee (0 \wedge 0) \\ (0 \wedge 0) \vee (1 \wedge 0) \vee (1 \wedge 1) & (0 \wedge 1) \vee (1 \wedge 0) \vee (1 \wedge 1) & (1 \wedge 0) \vee (1 \wedge 1) \vee (1 \wedge 0) \\ (1 \wedge 0) \vee (1 \wedge 0) \vee (1 \wedge 1) & (1 \wedge 1) \vee (1 \wedge 0) \vee (1 \wedge 1) & (1 \wedge 0) \vee (1 \wedge 1) \vee (1 \wedge 0) \end{bmatrix}$$

$$= \begin{bmatrix} 0 \vee 0 \vee 0 & 1 \vee 0 \vee 0 & 0 \vee 1 \vee 0 \\ 0 \vee 0 \vee 1 & 0 \vee 0 \vee 1 & 0 \vee 1 \vee 0 \\ 0 \vee 0 \vee 1 & 1 \vee 0 \vee 1 & 0 \vee 1 \vee 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Ex 12

Consider the equivalence relation from Example 2, namely, $R = \{(x, y) \mid x - y \text{ is an integer}\}$.

a) What is the equivalence class of 1 for this equivalence relation?

b) What is the equivalence class of 1/2 for this equivalence relation?

a) $R = \{(x,y) : x - y \in \mathbb{Z}\} \Rightarrow [1]_R = \{y : (1,y) \in R\} = \{y : 1 - y \in \mathbb{Z}\} = \mathbb{Z}$.

b) $R = \{(x,y) : x - y \in \mathbb{Z}\} \Rightarrow [1/2]_R = \{y : (\frac{1}{2}, y) \in R\} = \{y : \frac{1}{2} - y \in \mathbb{Z}\} = \{(2n+1)/2 : n \in \mathbb{Z}\}$.